

Cambridge International AS & A Level

CANDIDATE
NAME

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

[illegible]

- 3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \quad y = (t - 1)e^{-2t},$$

where $t > -2$.

- (a) Express $\frac{dy}{dx}$ in terms of t , simplifying your answer. [5]

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- (b) Find the exact y-coordinate of the stationary point of the curve. [2]

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4 Let $f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions.

[3]

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(b) Hence find $\int_1^2 f(x) \, dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

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- 5 (a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2} \tan \theta$ may be expressed as $\tan^4 \theta + 2 \tan^2 \theta - 7 = 0$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2} \tan \theta$, for $0^\circ < \theta < 180^\circ$.

[3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a)** By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leq \pi$. [2]

- (b)** Verify by calculation that this root lies between 1 and 1.5. [2]

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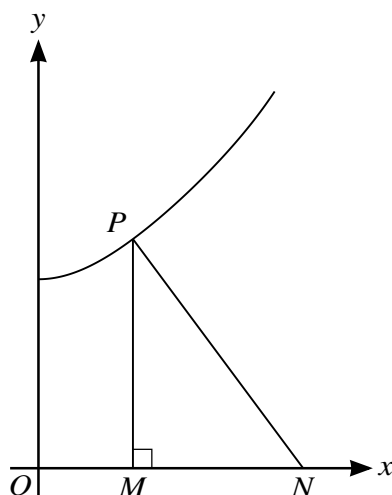
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- (c) Use the iterative formula $x_{n+1} = 2 \tan^{-1} \left(\frac{1}{1 + e^{-x_n}} \right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x -axis at N . The point M is the foot of the perpendicular from P to the x -axis.

The curve is such that for all values of x in the interval $0 \leq x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

- (a) (i) Show that $\frac{MN}{y} = \frac{dy}{dx}$. [1]

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- (ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2 \frac{dy}{dx} = \tan x$. [2]

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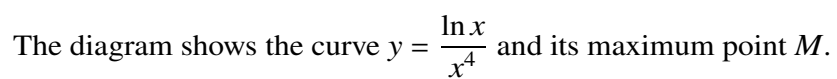
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- (b) Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing y in terms of x . [6]

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- [illegible]

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4} dx < \frac{1}{9}$. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 9 The quadrilateral $ABCD$ is a trapezium in which AB and DC are parallel. With respect to the origin O , the position vectors of A , B and C are given by $\vec{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\vec{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

- (a) Given that $\vec{DC} = 3\vec{AB}$, find the position vector of D . [3]

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- (b) State a vector equation for the line through A and B . [1]

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- (c) Find the distance between the parallel sides and hence find the area of the trapezium. [5]

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- 10 (a)** Verify that $-1 + \sqrt{2}i$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

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- (b)** Find the other roots of this equation. [7]

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